

Übungen zur Vorlesung
Effiziente kombinatorische Algorithmen
 WS 2019/20
 Blatt 6

AUFGABE 13:

Gegeben sei eine „leere“ $n \times n$ -Matrix M mit Zeilensummen a_i und Spaltensummen b_j . Die Aufgabe besteht nun darin, die Matrix mit Nullen und Einsen so zu füllen, dass sich die Summen dabei ergeben:

				3
				2
				2
				1
3	1	2	2	

1	0	1	1
1	0	0	1
0	1	1	0
1	0	0	0

0	1	1	1
1	0	1	0
1	0	0	1
1	0	0	0

Wie kann eine korrekte Füllung der Matrix – falls eine solche existiert – effizient durch Lösen eines geeigneten Flussproblems ermittelt werden?

AUFGABE 14:

You are helping the medical consulting firm DOCTORS WITHOUT WEEKENDS set up the work schedules of doctors in a large hospital. They've got the regular daily schedules mainly worked out. Now, however, they need to deal with all the special cases and, in particular, make sure that they have at least one doctor covering each vacation day.

Here's how it works. There are k vacation periods (e. g., the week of Christmas, the July 4th weekend, the Thanksgiving weekend, ...), each spanning several contiguous days. Let D_j be the set of days included in the j^{th} vacation period; we will refer to the union of all these days, $\bigcup_j D_j$, as the set of all *vacation days*.

There are n doctors at the hospital, and doctor i has a set of vacation days S_i when he or she is able to work. (This may include certain days from a given vacation period but not others; so, for example, a doctor may be able to work the Friday, Saturday, or Sunday of Thanksgiving weekend, but not the Thursday.)

Give a polynomial-time algorithm that takes this information and determines whether it is possible to select a single doctor to work on each vacation day, subject to the following constraints.

- For a given parameter c , each doctor should be assigned to work at most c vacation days total, and only days when he or she is available.
- For each vacation period j , each doctor should be assigned to work at most one of the days in the set D_j . (In other words, although a particular doctor may work on several vacation days over the course of a year, he or she should not be assigned to work two or more days of the Thanksgiving weekend, or two or more days of the July 4th weekend, etc.)

The algorithm should either return an assignment of doctors satisfying these constraints or report (correctly) that no such assignment exists.

Anmerkung: Thanksgiving ist in den USA immer am vierten Donnerstag im November. Es ist dort im An-denken an die Pilgerväter das wichtigste Familienfest überhaupt, zu dem man traditionell die Familie zum Truthahnessen besucht. Zudem markiert der Freitag nach Thanksgiving, der sog. Black Friday, den Beginn der Weihnachtszeit. Im übrigen läuft in der Zeit von Donnerstag bis Sonntag in den USA praktisch nichts.

Neben dem Valentinstag und Halloween hat nun mit dem Black Friday ein weiterer „besonderer“ Tag aus den USA Deutschland erreicht.