Synthesizing Passive Networks by applying Genetic Programming and Evolution Strategies

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Abstract- In order to accurately predict the behavior of micro-electronic systems with nodal based software tools like SPICE it is necessary to know appropriate equivalent circuits of the systems of interest. In microwave and RF engineering this equivalent networks often have to be derived form measurement or EM field calculation via scattering parameters.
In this contribution a methodology is suggested that combines Evolutionary Algorithms (EAs) with a nodal based assembly technique in order to synthesize passive equivalent networks. Using only the knowledge of the scattering parameters, both structure *and* component values of an equivalent circuit of a system are determined by EA.

1 Introduction

In order to apply nodal based simulation to micro-electronic systems containing passive elements like antennas, inhomogeneous micro-strip lines, integrated passives, vial holes etc. it is necessary to know equivalent circuit representations of these elements. Often the only information available is the devolution of the scattering parameter over frequency gained by electro-magnetic field calculation or measurements. While the optimization of the equivalent circuit element parameter is well covered by methods like gradient techniques, downhill simplex methods and evolutionary approaches like [Van00], [Wer00] and [Zhe03] the definition of the structure is typically left to the user requiring expert knowledge.
This paper presents a new technique to derive both structure *and* element parameters from given scattering parameters using Evolutionary Algorithms. In contrast to [Gri00] where equivalent networks are synthesized using Genetic Algorithms and [Koz96] suggesting deriving network structures using Genetic Programming this approach uses Evolutionary Strategies and a hybrid approach consisting of Genetic Programming and Evolutionary Strategies to accomplish this task.
Section 2 points out how single port and two port networks are represented in order to apply Evolutionary Algorithms. Also this section points out how the impedance, scattering parameters and the fitness is calculated.
In section 3 the algorithms are applied and tested with two examples.
Finally section 4 gives a conclusion on the work and highlights possible future development of this research.

2 Synthesis of Passive Networks

In this work two approaches are analyzed to synthesize passive networks. The first attempt highlights the synthesis of single port networks by using a combination of genetic programming and evolution strategy techniques. The second attempt shows how to synthesize two port networks by applying evolution strategies.

2.1 Single port networks

Figure 1 shows a general single port.

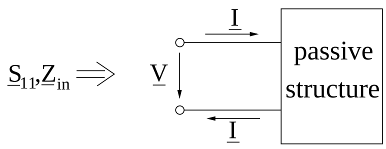


Figure 1: Schematic representation of a general *single port*

Synthesizing and optimizing a single port network is done by using a hybrid combination of genetic programming and evolution strategy techniques. The topology of the network is created and optimized by the genetic programming part. Meanwhile the evolutionary strategy optimizes the parameters of the used elements of the network. In order to use this approach the structure is mapped on a tree which consists of function nodes and terminal leafs. Feasible trees consist of up to four different types of functional nodes:

1. R: This node represents a resistor with the resistance R.
2. L: By this node an inductance with the inductivity L is allegorated.

3. C: A capacitor with capacity C is described by this node.
4. JUNCT: Represents a junction in order to shunt parts of the network.

Figure 2 shows the chosen function set. The operation necessary to compute the impedance of a network mapped to a tree is implemented by these nodes. The parameter $\underline{s1}$, $\underline{s2}$ and $\underline{s3}$ reference to data consisting of a complex value and a corresponding frequency f.

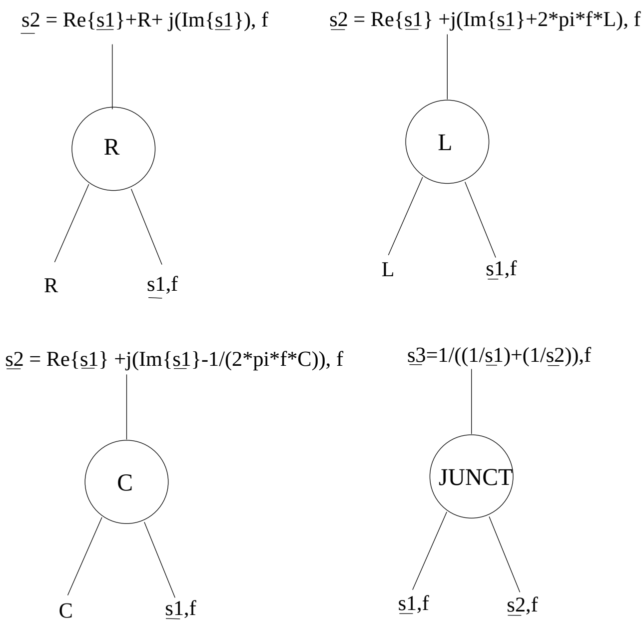


Figure 2: In order to represent a passive network by a tree the above functional nodes have been modeled

In figure 2, R, L and C denote real valued parameters of a resistor, inductivity and capacitor, respectively.
In order to reduce the space of feasible solutions it is possible to reduce the number of possible constellation of trees. For example a functional node can not be connected by the same node except of the JUNCT node. This is reasonable because the series connection of two equal elements can also be described with only one element having another parameter.
The terminal set needed by the genetic program consists of the following elements: Terminal Set $= \{U, \text{FinalDoubleTerminal}\}$. Here U represents an input signal and FinalDoubleTerminal refers to a real number which will be optimized by evolutionary strategies.

2.1.1 Impedance and Reflection Coefficient Evaluation

The desired result of this calculation is the impedance of the network sampled at different frequencies. In order to store this result it is pragmatic to set up a special data type. This data type is a linked list whose elements consist of a complex number with an associated frequency. The calculation of the impedance of a network can be demonstrated using a simple example as shown in figure 3.

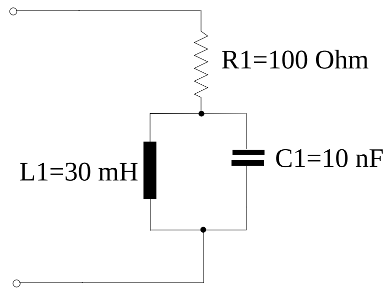


Figure 3: A simple example of a single port network

The mapping of the example network on a tree and the following calculation of the impedance is displayed in figure 4.

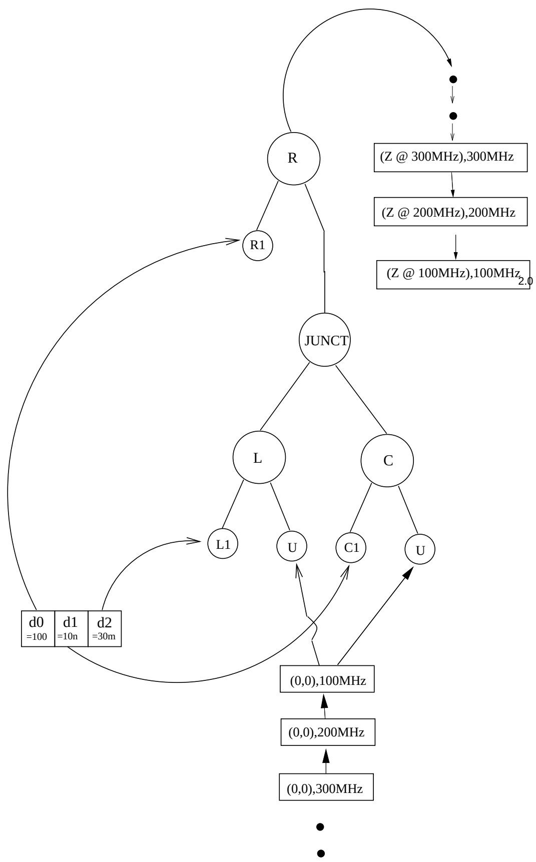


Figure 4: Calculation of the impedance of the network described in figure 3 by using a tree representation

The reflection coefficient ρ of a single port network is calculated by

$$\rho = \frac{Z/Z_w - 1}{Z/Z_w + 1} \quad (1)$$

where Z denotes the impedance of the network and Z_w is the characteristic impedance (typically 50 Ω).

2.1.2 Fitness Evaluation

The aim of the synthesis of passive networks here is to generate a network which has a desired devolution of the (complex) reflection coefficient. In order to find such a network the fitness of the individuals has to be proportional to the aberration of the calculated reflection coefficient to the desired reflection coefficient. A proper fitness function is defined via

$$F = \sum_{n=1}^{N} \left|\rho_{d_n} - \rho_{c_n}\right|^2 \frac{\Delta f_n}{\Delta F} \quad \text{with}$$
$$\Delta f_n = \begin{cases} |f_1 - f_2| & \text{for } n = 1 \\ \frac{|f_{n-1} - f_{n+1}|}{2} & \text{for } 1 < n < N \\ |f_N - f_{N-1}| & \text{for } n = N \end{cases} \tag{2}$$
$$\Delta F = f_N - f_1$$

Here ρ_{d_n} refers to the desired reflection coefficient at the n'th frequency point. Similar ρ_{c_n} indicates the computed reflection coefficient of the actual individual. By this definition of the fitness function it is not necessary to sample the reflection coefficient at equally spaced frequency points.

2.2 Two-Port Networks

Generally a two port network can be depicted as shown in figure 5.
Because a suitable function set couldn't be found to calculate the impedance of two port networks a different approach has been chosen.

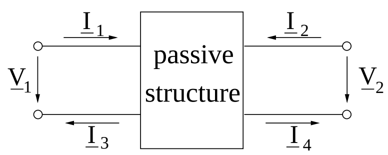


Figure 5: Demonstration of a two port

In order to synthesize and optimize two port networks evolutionary strategies have been chosen. To apply this class of algorithms a network is described by vectors. Here the composition of a network is mapped to three vectors. Two of these vectors supply the structural information of the network and the third vector describes the component values of the elements used inside the network.

2.2.1 Describing Two Port Networks using Vectors

To describe the composition of a network we use the so called incidence matrix of the network. This matrix indicates which node is connected to which node via a certain branch. The convention used in this work can be described by the following example.
Figure 6 shows a simple two port network. The internal nodes are emphasized.

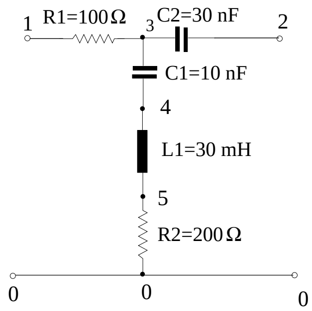


Figure 6: Example of a two port network

The incidence matrix of this network is:

$$\overline{\overline{I}} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \tag{3}$$

The rows of the matrix refer to the branches and the columns procure to the nodes. Here branch 0 indicates a connection between node 1 and node 3 etc. Node 0 always represents the reference node (also called ground node). All voltages inside the network refer to this node. Node 1 and 2 indicate the external nodes of the two-port. If the network consists of n internal nodes and k branches then the incidence matrix has k rows and n+1 columns [1].
Due to the fact, that for typical networks the incidence matrix has only a few non-zero elements, this matrix can be compressed by describing the network with just two numbers per column indicating the two nodes the branch is connected between. Applying this to the matrix I from equation (3) leads to

$$\overline{\overline{I}}_{comp} = \begin{bmatrix} 1 & 3 \\ 3 & 4 \\ 4 & 5 \\ 5 & 0 \\ 2 & 3 \end{bmatrix} \tag{4}$$

This matrix has to be mapped to a vector in order to optimize it using evolutionary strategies. This mapping is done by

[1] The ground node is also counted.

$$\overline{I}_{vect} = \begin{pmatrix} I_{comp_{0,0}} \\ I_{comp_{0,1}} \\ I_{comp_{1,0}} \\ I_{comp_{1,1}} \\ \vdots \\ I_{comp_{n,0}} \\ I_{comp_{n,1}} \end{pmatrix} \quad (5)$$

The incidence matrix from equation (3) now becomes

$$\overline{I}_{vect}^T = (1, 3, 3, 4, 4, 5, 5, 0, 2, 3) \quad (6)$$

In order to describe the structure of a passive network fully it is necessary to provide additional information about the type of elements used. This information is stored in a second vector T whose rows correspond to the branches of the network. For this example the vector is

$$\overline{T}^T = (R, C, L, R, C) \quad (7)$$

In order to map the elements of the vector to integers the mapping

$$R := 0; \qquad L := 1; \qquad C := 2 \quad (8)$$

has been chosen. By applying this to equation (7) we obtain

$$\overline{T}_{int}^T = (0, 2, 1, 0, 2) \quad (9)$$

The parameters of the elements are stored in a third vector consisting of real numbers. By previous knowledge the search space can be confined by assuming

$$\begin{array}{l} R \in (0\Omega, R_{max}] \\ L \in (0H, L_{max}] \\ C \in (0F, C_{max}] \end{array} \quad (10)$$

This assumption affects the initialization. During the process of optimization the parameters can exceed these values. In order to get reasonable results when applying initializing and recombination it is necessary that all elements of this vector are in the same value range.

The parameters of a network consisting of n branches can be represented by using a vector $\overline{x_p}$ with the elements $\overline{x_{p_i}} \in (0, 1] \forall i \in \{1, 2..., n\}$. By using the normalizing factors $\text{NORM}_R = R_{max}, \text{NORM}_L = L_{max}, \text{NORM}_C = C_{max}$ and information about the structure of the network the elements of the parameter vector can be mapped back to a sensible value. In case of the example from figure 6 the value vector is

$$\overline{x_p} = \begin{pmatrix} 0.1 \\ 0.01 \\ 0.03 \\ 0.2 \\ 0.03 \end{pmatrix} \quad \text{with} \quad \begin{array}{r} R_{max} = 1k\Omega \\ L_{max} = 1H \\ C_{max} = 1mF \end{array} \quad (11)$$

with aid of equation (7) the parameter vector can be determined:

$$\overline{p} = \begin{pmatrix} 100\Omega \\ 10nF \\ 30mH \\ 200\Omega \\ 30nF \end{pmatrix} \quad (12)$$

2.2.2 Computing impedance and scattering parameters

In order to designate the admittance matrix all nodes of the network of interest are interpreted as accessible from outside. Only node 0 is treated different as mentioned above. A network consisting of $n+1$ nodes[2] features therefore an admittance matrix with n-rows and n columns. From [Kel01] follows that this network can be interpreted as a composition of many Π-networks as shown in figure 7.

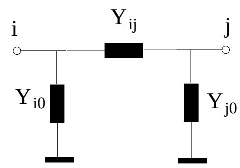


Figure 7: Illustration of a Π-network

By applying the modified nodal based assembly technique as described in [Haa98] the admittance matrix can be composed using the following method:

First the Y-Matrix is initialized by setting up all entries of the matrix to $\underline{Y_0} = 0 + j0$.

Then, the following schema is applied:

$$\begin{array}{rcl} Y_{ii_{new}} & = & Y_{ii_{old}} + Y_{ij} + Y_{i0} \\ Y_{ij_{new}} & = & Y_{ij_{old}} - Y_{ij} \\ Y_{ji_{new}} & = & Y_{ji_{old}} - Y_{ij} \\ Y_{jj_{new}} & = & Y_{jj_{old}} + Y_{ij} + Y_{j0} \end{array} \quad (13)$$

Now it is possible to compute the Z-parameters from this admittance matrix. This is done by solving the following equation system for $\overline{V}$:

[2]this includes node 0

$$\overline{I} = \overline{\overline{Y}} \cdot \overline{V} \quad \text{that is}$$

$$\begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{pmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{pmatrix} \quad (14)$$

Here $\overline{V}$ is a vector constituting the potentials referred to ground at all nodes of the network.
$\overline{I}$ is the current vector which has to fulfill boundary conditions which will be discussed in the following.
Because

$$Z_{11} = \left.\frac{U_1}{I_1}\right|_{I_2=0} \text{ and } Z_{21} = \left.\frac{U_2}{I_1}\right|_{I_2=0} \quad (15)$$

$\overline{I}$ has to look like

$$\overline{I}^T = (1 + j0, 0, 0, \cdots, 0) \quad (16)$$

in order to compute Z_{11} and Z_{21}. Now Z_{11} and Z_{21} can be gained directly from $\overline{V}$ by solving equation (14).
In the same way Z_{22} and Z_{12} maybe computed using the relation

$$Z_{22} = \left.\frac{U_2}{I_2}\right|_{I_1=0} \text{ and } Z_{12} = \left.\frac{U_1}{I_2}\right|_{I_1=0} \quad (17)$$

and

$$\overline{I}^T = (0, 1 + j0, 0, \cdots, 0) \quad (18)$$

In order to compute the scattering parameters the Z-parameters have to be normalized with the characteristic impedance Z_w.

$$\begin{array}{ll} z_{11} = \frac{Z_{11}}{Z_w} & z_{12} = \frac{Z_{12}}{Z_w} \\ z_{21} = \frac{Z_{21}}{Z_w} & z_{22} = \frac{Z_{22}}{Z_w} \end{array} \quad (19)$$

The relationship between Z-Parameters and S-Parameters is finally obtained according to [Bal81]

$$\overline{\overline{S}} = (\overline{\overline{Z}}_n - \overline{\overline{U}})(\overline{\overline{Z}}_n + \overline{\overline{U}})^{-1} \text{ with}$$
$$\overline{\overline{U}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } \overline{\overline{Z}}_n = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} \quad (20)$$

In case of a two port this simplifies to

$$\begin{array}{ll} S_{11} = \frac{\Delta_z + z_{11} - (z_{22}+1)}{\Delta_z + z_{11} + z_{22} + 1} & S_{12} = \frac{2z_{12}}{\Delta_z + z_{11} + z_{22} + 1} \\ S_{21} = \frac{2z_{21}}{\Delta_z + z_{11} + z_{22} + 1} & S_{22} = \frac{\Delta_z + z_{22} - (z_{11}+1)}{\Delta_z + z_{11} + z_{22} + 1} \end{array} \quad (21)$$

With $\Delta_z = det(z) = z_{11}z_{22} - z_{12}z_{21}$.

2.2.3 Implementing the fitness function

The calculation of the fitness is analog to equation (2) with the extension that here 4 scattering parameters have to be taken into account.

$$F = \sum_{n=1}^{N} \sum_{i=1}^{2} \sum_{j=1}^{2} \left| S_{ijd_n} - S_{ijc_n} \right|^2 \frac{\Delta f_n}{\Delta F} \quad (22)$$

Here S_{ijd_n} refers to the desired scattering parameter ij at the n'th frequency point and S_{ijc_n} is the computed scattering parameter at this frequency. The remaining variables can be found in equation (2).

3 Case Study

In order to demonstrate the performance of genetic programming and evolutionary strategies to synthesize passive network a software tool has been developed which implements the synthesis of passive single ports and two ports.
For mutating the parameter vector the algorithm from [Deb99] has been chosen:

$$\tilde{x_j} = x_j + \delta_{max} \cdot \overline{\delta_j} \qquad \forall j \in \{1, 2, \ldots, n\} \quad (23)$$

Here x_j is the j'th component of the original parameter vector and $\tilde{x_j}$ is the j'th component of the altered parameter vector. The strategic parameter δ_{max} is setting up the maximum possible change of the components of the vector. $\overline{\delta_j}$ is computed by

$$\overline{\delta_j} = \begin{cases} (2U_j)^{\frac{1}{k+1}} - 1 & \forall U_j < 0,5 \\ 1 - (2(1-U_j))^{\frac{1}{k+1}} & \forall U_j \geq 0,5 \end{cases} \quad (24)$$
$$j \in \{1, 2, \ldots, n\}$$

U_j is a uniform distributed random variable in the interval $[0, 1)$. Again k is a strategic parameter. Experiment have shown that using $k = 4$ results in the best convergence of an EA using this mutation algorithm.

3.1 Example 1: Input impedance of a dipole antenna

In this example the equivalent network of a single port dipole antenna is synthesized. The reflection coefficient S_{11} of this antenna computed by a program which is solving the Maxwell equations is shown in figure 10. It is denoted with $S11_{comp}$.
It can be obtained that the magnitude of the reflection coefficient is very small in the range of 1.2 G-Hz. At this frequency more than 85% of the delivered energy are absorbed respectively radiated into space. The phase is declining monotonously. The seeming phase skipping at 1.2 G-Hz is leading from the ambiguity of the a-tan function.

3.1.1 Parameter Settings

The synthesizing is done by using a hybrid algorithm as described in section 2.1. The following parameters have been chosen:

Population size	:100
Mutation probability	:0.2 (tree),0.3 (parameter)
Crossover probability	:0.7 (tree),0.5 (parameter)
δ_{max}	:20%
R_{max}	:10 Ω
L_{max}	:10 nH
C_{max}	:1 nF
max tree depth during initialization	:5
max tree depth during optimization	:10
max # of edges during initialization	:10
max # of edges during optimization	:30

3.1.2 Results

In order to test the reproducibility of the algorithm for this example 20 runs have been performed. Here the algorithm was stopped after 300 generations for each run. The fitness value for the runs is displayed in figure 8. The worst possible fitness value is 4.0 and the best fitness value is 0.

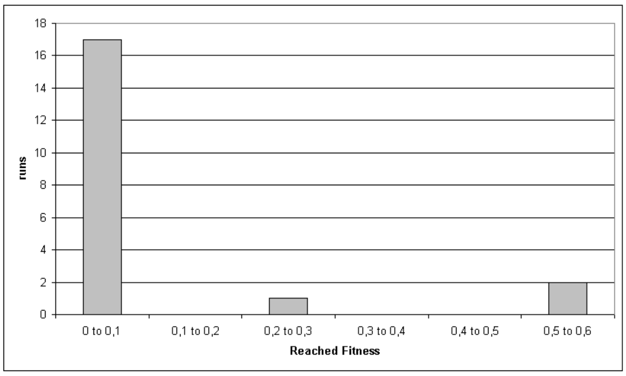


Figure 8: The resulting fitness values for different runs

The best solution which is presented by run 11 is shown in figure 9

This network leads to the reflection coefficient as shown in figure 10.

The development of the fitness is demonstrated in figure 11.

3.2 Example 2: Micro-strip discontinuity

In this example an equivalent circuit for a micro-strip line on top of a defected ground plane is searched. The given structure is shown in figure 12.

The scattering parameters for this structure have been calculated using a 3D EM field solver. They refer to the both ends of the micro-strip line. The scattering parameter are denoted in figure 13 by S_{des}.

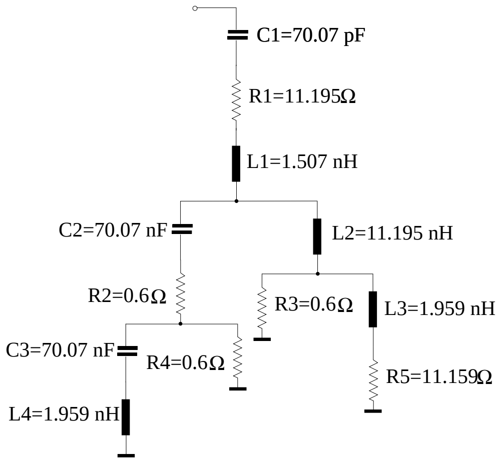


Figure 9: The equivalent network of the best solution presented by the algorithm

3.2.1 Parameter Settings

In this case the synthesis is done by evolutionary strategies as described in 2.2. The following parameters have been chosen:

Population size	:100
Mutation probability	:0.05 (matrix),0.05 (type),0.05 (value)
Crossover probability	:0.01 (matrix),0.01 (type),0.01 (value)
δ_{max}	:90%
R_{max}	:1000 Ω
L_{max}	:10 nH
C_{max}	:1 pF
max # of nodes	:4
# of branches	:8

3.2.2 Results

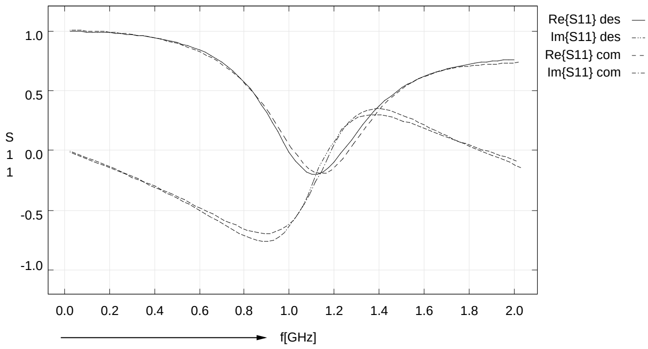
The scattering parameters calculates of one suggestion the algorithm presented is displayed in figure 13 and 14

Because the network is passive it is imperative that $S_{12} = S_{21}$ as shown in [Bal81]. The found network is shown in figure 15

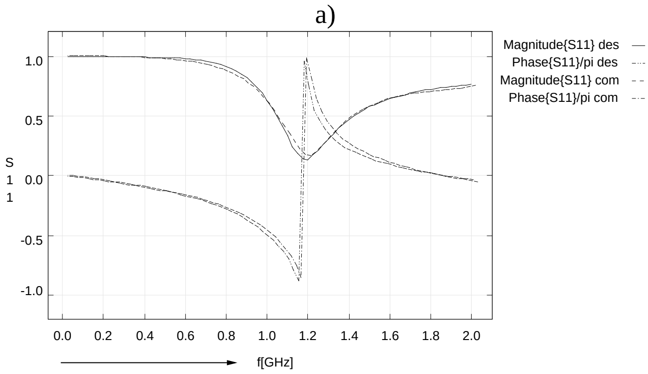
4 Conclusion and Future Work

In this paper it has been pointed out how single port networks can be represented using a tree representation and it has been shown how to map two port network to a set of vectors. This enabled the application of genetic programming and evolutionary strategies in order to synthesize passive networks. The process of synthesizing networks has been tested and validated with two examples.

Consequently it is now possible to generate equivalent network only with knowledge of the devolution of the reflection coefficient or the scattering parameters, without using a priori expert knowledge of the network structure. This is



a)



b)

Figure 10: The desired refection coefficient $S11_{des}$ and the suggestion of a solution by run 11 ($S11_{com}$). In a): Demonstration of the real and imaginary part and b) showing magnitude and phase. Also here a very good agreement between the synthesized model and the simulation results can be seen.

an important step towards an automated design process.
In the future, we would like to show more experiments with respect to non-hybrid representations and other techniques. Amazingly, our approach was able to obtain very good solutions for structure *and* parameters of passive networks.

Acknowledgment

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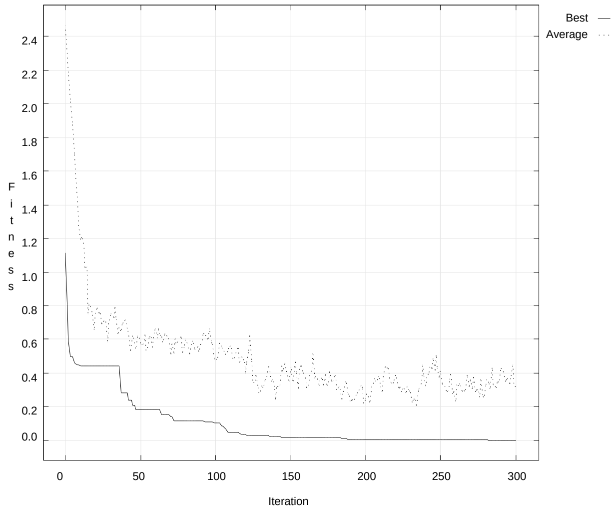

Figure 11: The progress of the fitness for run 11.

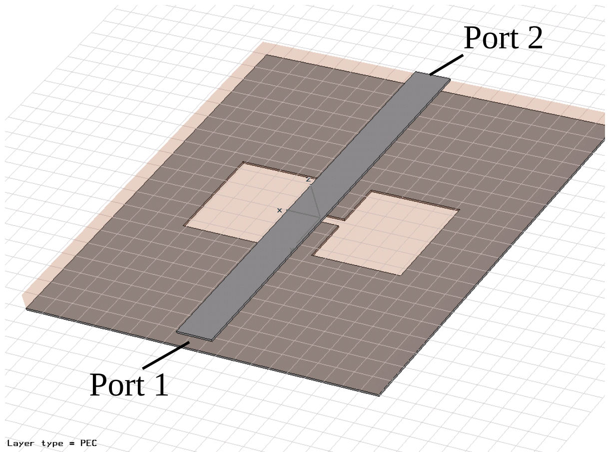

Figure 12: Micro-strip line on top of a defected ground plane

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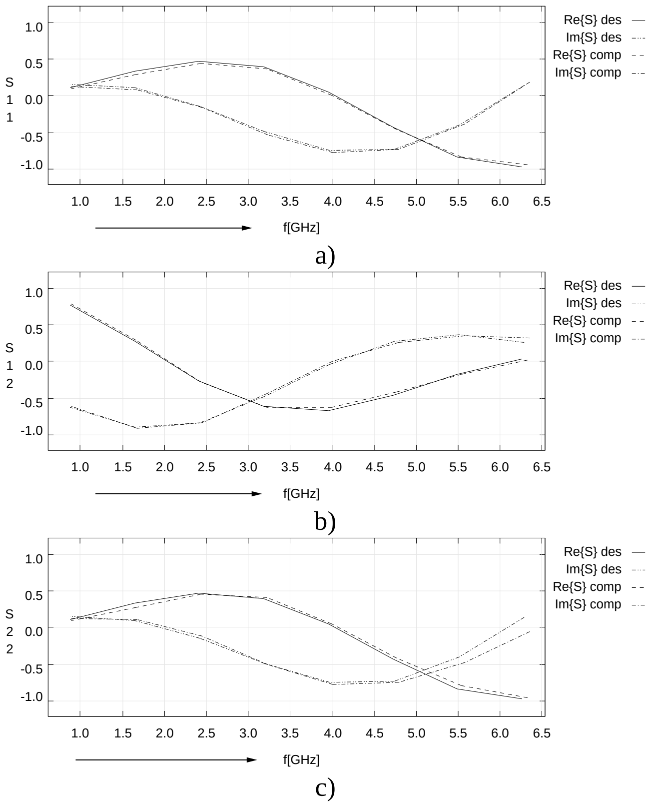

Figure 13: Real- and imaginary part of the desired refection coefficient Sij_{des} and the suggestion of a solution (Sij_{com}). In a): Demonstration of S_{11} b) showing S_{12} and c) displays S_{22}

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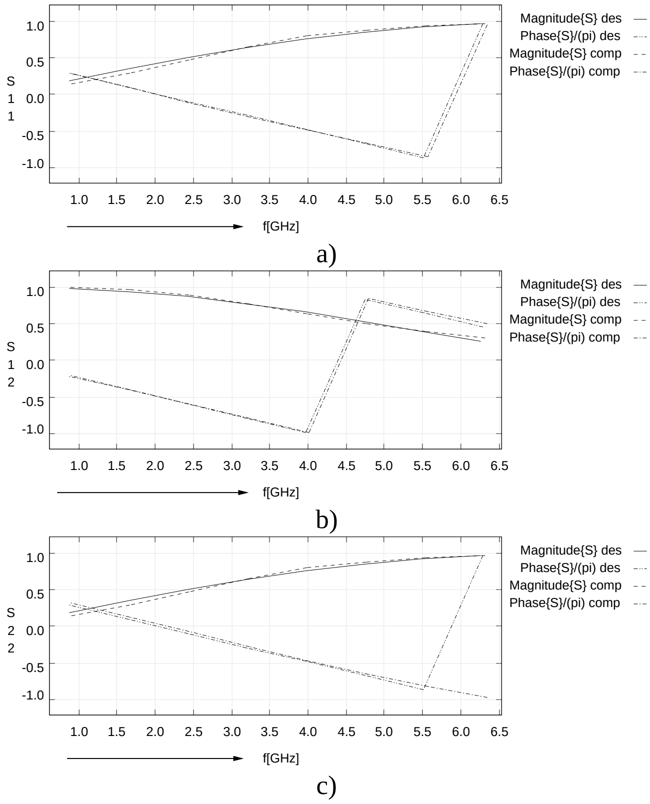

Figure 14: Magnitude and phase of the desired and the computed scattering parameters. In a): Demonstration of S_{11} b) showing S_{12} and c) displays s_{22}

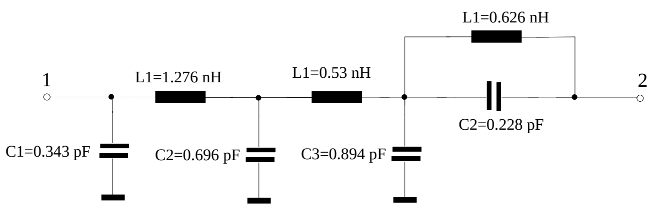

Figure 15: The presented equivalent network